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1984 J. Phys. A: Math. Gen. 17 245

(http://iopscience.iop.org/0305-4470/17/1/024)

J. Phys. A; Math. Gen. 17 (1984) 245–246. Printed in Great Britain

COMMENT

A short note on directed self-avoiding walks

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Received 7 June 1983, in final form 3 August 1983

Abstract. The critical behaviour of partially directed self-avoiding walks (SAWs) on a cubic lattice with only one axis directed is studied using exact enumerations. The critical exponents ($\nu_{\parallel}=0.999\pm0.005$, $\nu_{\perp}=0.501\pm0.006$ and $\gamma=1.000\pm0.001$) found here further support the universality hypothesis that various partially directed SAWs belong to the same universality class as fully directed SAWs and they are also dimension independent for $d\geqslant 2$.

In the past few years, systems with directionally dependent critical behaviour have been the focus of much attention. The most studied systems are directed percolation (see e.g. Kinzel 1983) and directed lattice animals (see e.g. Redner and Yang 1982). Recently, Nadal *et al* (1982) have pointed out that any fully directed SAW of N steps can be decomposed into a directed walk along the symmetry axis and a random walk perpendicular to the symmetry axis. The corresponding critical exponents $\nu_{\parallel}$ and $\nu_{\perp}$ for the root-mean-square end-to-end displacements, $\langle R_{\parallel}^2(N)\rangle \sim N^{2\nu_{\parallel}}$ (or $\langle R_{\parallel}(N)\rangle \sim N^{\nu_{\parallel}}$) and $\langle R_{\perp}^2(N)\rangle \sim N^{2\nu_{\perp}}$, have the values $\nu_{\parallel}=1$ and $\nu_{\perp}=\frac{1}{2}$. These are true for any dimension $d\geqslant 2$. Let $G(N)$ be the total number of walks in N steps. In general, when N is large, $G(N)$ can be expressed in the form $G(N)\sim \mu^N N^{\gamma-1}$ (McKenzie 1976) where μ is the connectivity constant and γ corresponds to the susceptibility exponent of the spin system. For the fully directed SAWs, it is easy to see that $G(N)=d^N$ which gives $\mu=d$ and $\gamma=1$. According to the universality concept, we would expect that the above values for $\nu_{\parallel}$, $\nu_{\perp}$ and γ are also true for any partially directed SAW in dimension $d\geqslant 2$.

Partially directed SAWs in two dimensions were first studied by Chakrabarti and Manna (1983). Redner and Majid (1983), using the transfer matrix method, were able to obtain exactly the same exponents as fully directed SAWs for those systems in which closed loops are not allowed because of the directed walks (e.g. some two-dimensional partially directed SAWs and d-dimensional directed SAWs with $d-1$ axes directed).

In this short note, we have performed the exact enumerations of partially directed SAWs in a cubic lattice with only one axis directed. The results from these enumerations are shown in table 1. The ratio method is used to estimate μ and γ from $G(N)$. From the sequence $b_N=G(N)/G(N-1)$, the values of μ and γ are estimated from the linear extrapolation of $b_N\sim\mu[1+(\gamma-1)/N]$. We find $\mu=4.160\pm0.001$ and $\gamma=1.000\pm0.001$. To estimate the values of $\nu_{\parallel}$ and $\nu_{\perp}$, we first examine the dependence of $\nu_{\parallel}(N)$ and $\nu_{\perp}(N)$ versus N on a double logarithmic scale. The values of $\nu_{\parallel}(N)$ and $\nu_{\perp}(N)$ are calculated from the slopes of successive data points (table 1). The limiting values of $\nu_{\parallel}$ and $\nu_{\perp}$ are extrapolated from $\nu_{\parallel}(N)$ and $\nu_{\perp}(N)$ sequences by means of

Table 1. Results for partially directed SAWs on a cubic lattice with only one axis directed.

N	$G(N)$	$(\langle R_{\parallel}^2(N)\rangle)^{1/2}$	$(\langle R^2(N)\rangle)^{1/2}$	b_N	$\nu_{\parallel}(N)$	$\nu_{\perp}(N)$
1	5	0.4472	0.8944	—	—	—
2	21	0.7559	1.3801	4.2	0.7573	0.6258
3	89	1.0440	1.7482	4.2381	0.7962	0.5830
4	369	1.3414	2.0771	4.1461	0.8715	0.5992
5	1 541	1.6337	2.3585	4.1762	0.8834	0.5694
6	6 405	1.9301	2.6167	4.1564	0.9143	0.5700
7	26 673	2.2249	2.8503	4.1644	0.9221	0.5548
8	110 921	2.5215	3.0685	4.1586	0.9372	0.5524
9	461 549	2.8175	3.2717	4.1611	0.9426	0.5442
10	1 919 765	3.1143	3.4636	4.1594	0.9506	0.5412

Neville tables. We find $\nu_{\parallel} = 0.999 \pm 0.005$ and $\nu_{\perp} = 0.501 \pm 0.006$. These results further support the universality hypothesis that various partially directed SAWs belong to the same universality class as fully directed SAWs and they are also dimension independent for $d \geq 2$.

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